THE INVERSE PROBLEM OF MAGNETOENCEPHALOGRAPHY: SOURCE LOCALIZATION AND THE SHAPE OF A BALL

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1. Intoduction

Magnetoencephalography (MEG) is a non-invasive neurophysiological technique that measures the magnetic fields generated by neuronal activity of the brain. The spatial distributions of the magnetic fields are analyzed to localize the sources of the activity within the brain, and the locations of the sources are superimposed on anatomical images, such as MRI, to provide information about both the structure and function of the brain.

Neural current sources in the brain produce external magnetic fields and scalp surface potentials that can be measured using magnetoencephalography (MEG).

Magnetic fields are found whenever there is a current flow, whether in a wire or a neuronal element.

The fundamental advantages of MEG are

- it measures brain function,
- high precision-millimeter resolution,
- high temporal resolution-millisecond resolution (it captures eliptic spikes),
- it is adaptable to mapping many function-sensory, motor, language, memory cortex,
- it is nonivasive and
- easy to use.

2. How Does MEG Work?

Magnetoencephalography (MEG) takes advantage of the fact that all electrical currents, be they in power lines, or brain cells, generate a surrounding magnetic field. Using special sensors it is possible to measure the tiny magnetic signals produced by the brain, even though these are more than one billion times smaller than the magnetic field generated by a light bulb. Different parts of the brain produce different patterns of magnetic waves. When the brain has been affected by disease, abnormal magnetic signals may be produced, and MEG can be used to determine which brain regions are malfunctioning. MEG can also be used to identify specific functional regions of the brain such as auditory and visual cortex. When a patient is presented with a stimulus (for example, a sound or a picture), specific portions of the brain are normally activated in characteristic sequence. By examining how the neuromagnetic activity changes during stimulation, it is possible to pinpoint the location of functional regions and to determine if the sequence of activation has been perturbed by disease.

3. The Inverse Problem of MEG

The inverse problem of magnetoencephalography (MEG) is to estimate impressed currents from observations of magnetic fields outside the skull. A common way to model impressed currents is to generate a grid that covers the region of interest in the brain and attach mutually orthogonal electric dipoles with unknown amplitudes at each grid point.

The MEG inverse problem of reconsructing electrical sources in human brain is an ill-posed problem. Users of MEG are faced with a vast array of inverse methods that can be used to process their data.For solving this problem there are some regularization techniques (i.e., Tikhonov regularization).

3.1. Tikhonov Regularization

A cortical map is computed by fitting the measured data in a least-squares sense. We use Tikhonov regularization to produce a stable solution with regularization parameter λ. We then normalize the minimum norm map to produce a statistic that gives uniform spatial specificity in the absence of activation.

We have a discretized model $L\alpha = b + e$. [5]

α : vector containing the dipole amplitudes α_j
b : vector containing the measured magnetic field components at the magnetometers
e : vector of observation noise
L : "lead field matrix" of the type $m \times n$ $(m \geq n)$ and with full rank.

The "minimum-norm solution" of MEG is commonly formulated as

$$\alpha_{MN} = \arg\min\{\|\alpha\| \,|\, \|L\alpha - b\| \leq d\};$$

it aims a *smallest comlexity* (*biggest stability*) under the given tolerance. Indeed, let *d* > 0 be an approximate tolerance of the error *e* , and α_{MN} be a solution of a constrained optimization problem:

$$\begin{cases} \min \|\alpha\|_2 \\ \text{subject to} \|L\alpha - b\|_2 \leq d. \end{cases}$$

It gets *stabilized* ("regularized") by using ***Tikhonov regularization*** technique which is perhaps the most widely used technique for regularizing discrete ill–posed problems. We paid a price for this stability in that the regularized solution had reduced resolution and was no longer unbiased. [1]:

$$(P)_\lambda \qquad \min_{\alpha} \|L\alpha - b\|_2^2 + \lambda^2 \|\alpha\|_2^2,$$

which means a least-squares problem with a *penalization term*. This problem can be solved by studying the normal equations. Using ***singular value decomposition*** (*SVD*):

$$L = USV^T,$$

we can find a unique solution

$$\alpha_\lambda = \sum_{i=1}^{k} \frac{s_i^2}{s_i^2 + \lambda^2} \frac{(U_{.,i})^T b}{s_i} V_{.,i} \qquad (k := \min\{m, n\})$$

with the ***filter factors*** $f_i = \frac{s_i^2}{s_i^2 + \lambda^2}$ [1].

When plotted on a log–log scale, the curve of optimal values of $\| \alpha \|_2$ versus $\| L\alpha - d \|_2$ often takes on a characteristic L shape. This happens because $\| \alpha \|_2$ is a strictly decreasing function of λ and $\| L\alpha - d \|_2$ is a strictly increasing function of λ. The sharpness of the "corner" varies from problem to problem, but it is frequently well–defined. For this reason, the curve is called an ***L–curve***. In addition to the discrepancy principle, another popular criterion for picking the value of λ is the L–curve criterion in which the value of λ that gives the solution closest to the corner of the L–curve is selected [1].

We can use the ***MATLAB regularization toolbox*** [http://www.mathworks.com] for performing Tikhonov regularization [6].

Firstly we compute the L-curve and find its corner which represents the optimal combination of our two goals, associated with the optimal λ and the corresponding solution of our problem $(P)_\lambda$ [1].

4. Simulation

It is known that for realistic noise levels the performance of both unimodal and bimodal systems do not improve with an increase in the number of measurements beyond approximately 100 [2]. Therefore the sensor number is chosen not so much than this number. An array of 124 MEG sensor is placed above the homogenous spherical head model (Figure 1). The sensor array is placed to cover only the upper hemisphere of the model. Sensor locations are noted below in the following illustration.

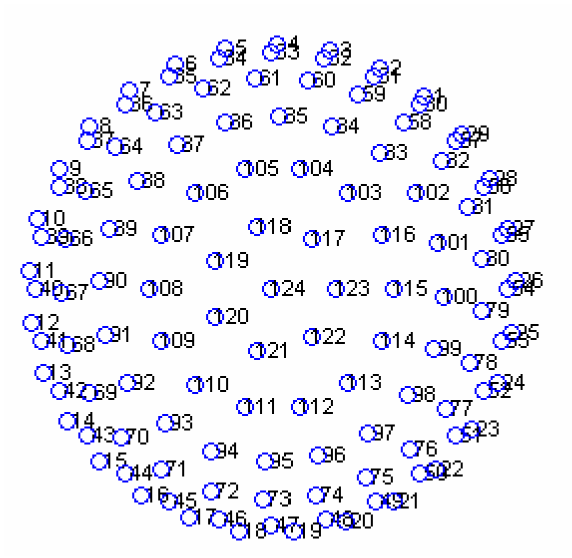

Figure 1: An array of 124 MEG sensor is placed above the homogenous spherical head model.

The radial magnetic field of the current dipole measured from a sensor is found as [4]

$$b_r(r) = \frac{\mu_0}{4\pi r d^3} \vec{r} \times \vec{r}_q \cdot \vec{q},$$

where r and r_q are the sensor location and dipole location, respectively, and q is the dipole orientation. Furthermore, d is the distance between observation and source point and r is the distance of the sensor to origin.

In this simulation, it is assumed to find neurophysiological activities on the cortex. Therefore, the dipole locations are chosen 1 cm below from the scalp surface. In this simulation, the simulated data are used to obtain results.

4.1 Single Dipole Case

A tangential current activity at the (8,0,0 cm) is assumed and the resultant measured magnetic fields, ($x10^{-7}$) are shown below (Figure 2).

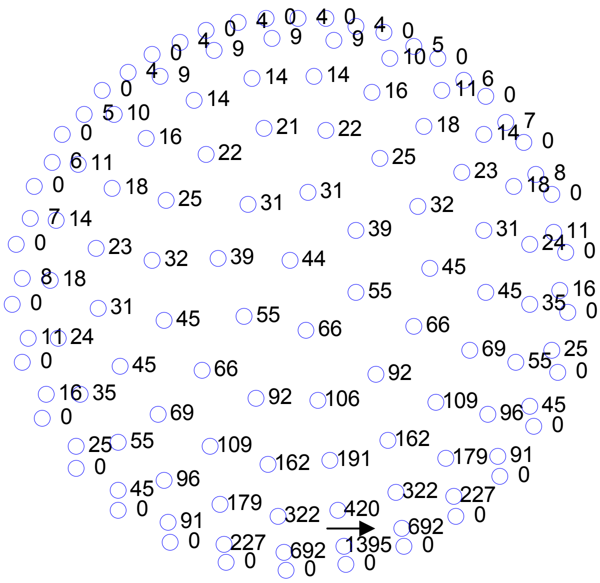

Figure 2: The resultant measured magnetic fields, ($x10^{-7}$) of the single dipole.

The L-curve of the single dipole (Figure 3) located at (8,0,0 cm) (Figure 2) and the corresponding solution (Figure 4) for that corner point is shown below. This case was considered to be noise free. If an additional noise 2dbW power is added to the measurement data the following L-curve (Figure 5) and solution (Figure 6) is also given.

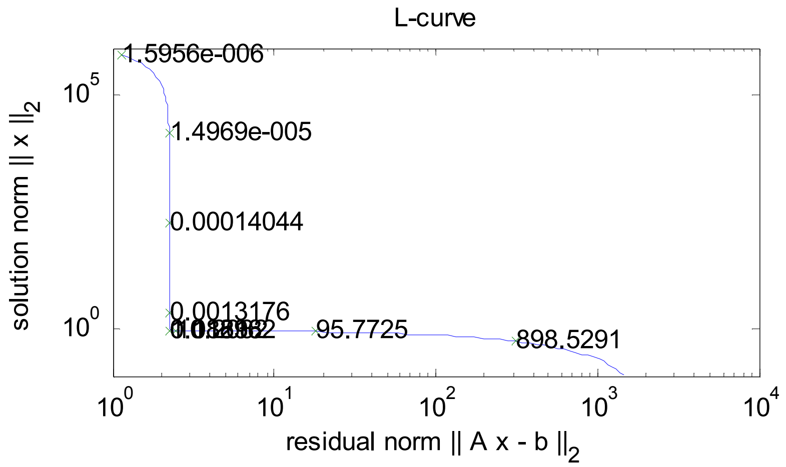

Figure 3: L-curve of the single dipole solution.

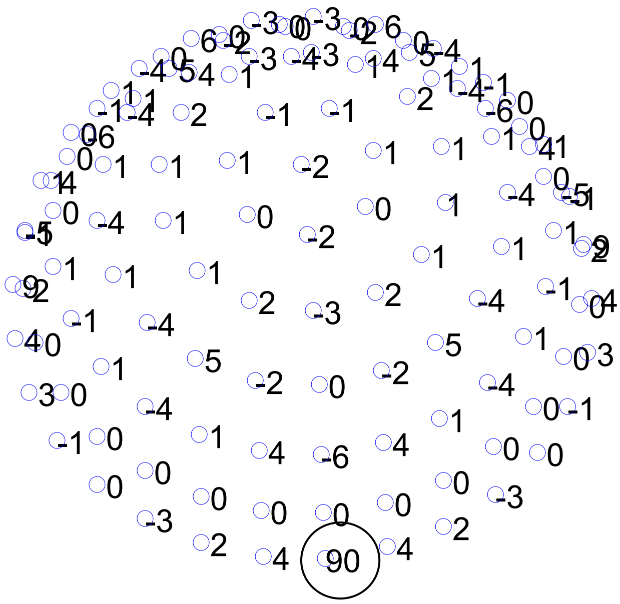

Figure 4: Tikhonov solution of the single dipole activity. The current activity is found at the expected region at the cortex and is highlighted by a circle.

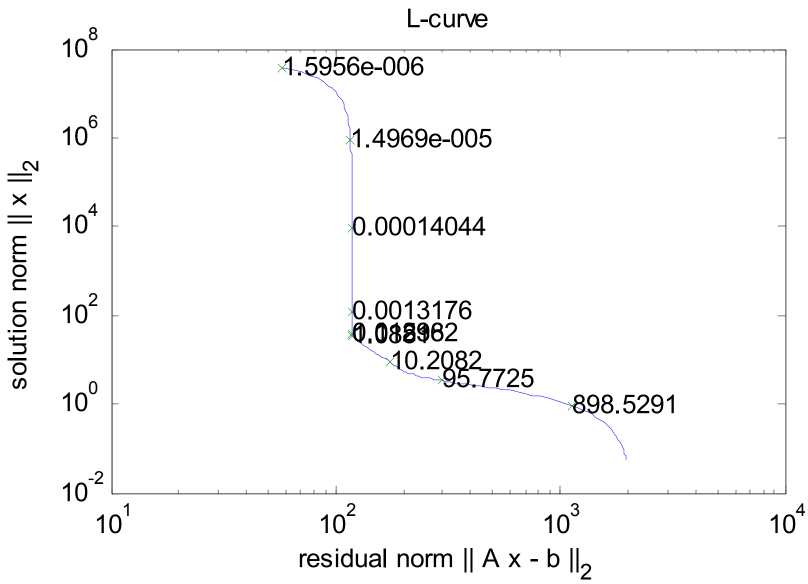

Figure 5: L-curve of the single dipole solution with an additional 2dbW white noise is added to measurements.

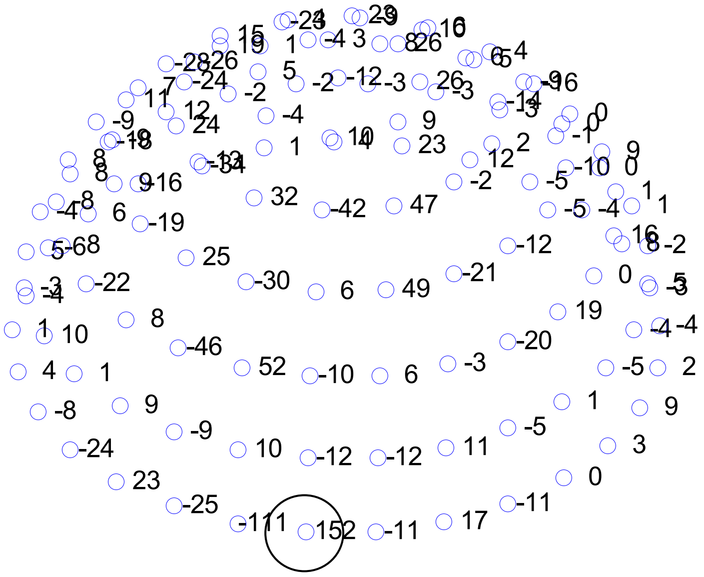

Figure 6: Tikhonov solution of the single dipole activity with an additional 2dbW white noise is added to measurements. The current activity is found at the expected region at the cortex and is highlighted by a circle in the figure.

4.2 2-Dipole Case: Opposite Dipoles

In that case two tangential current activities at the (8,0,0 cm) and (-8,0,0 cm) is assumed. The corresponding L-curve (Figure 7) and solution (Figure 8) are given below.

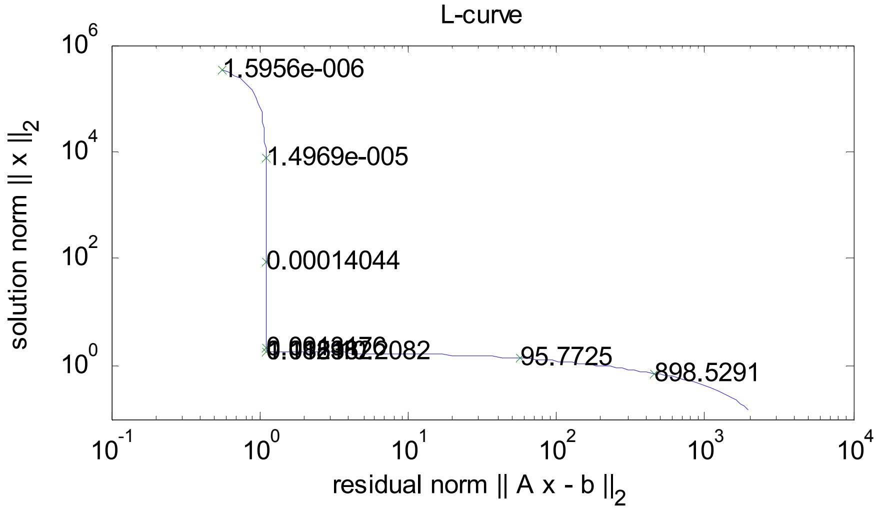

Figure 7: L-curve of the double opposite dipoles solution.

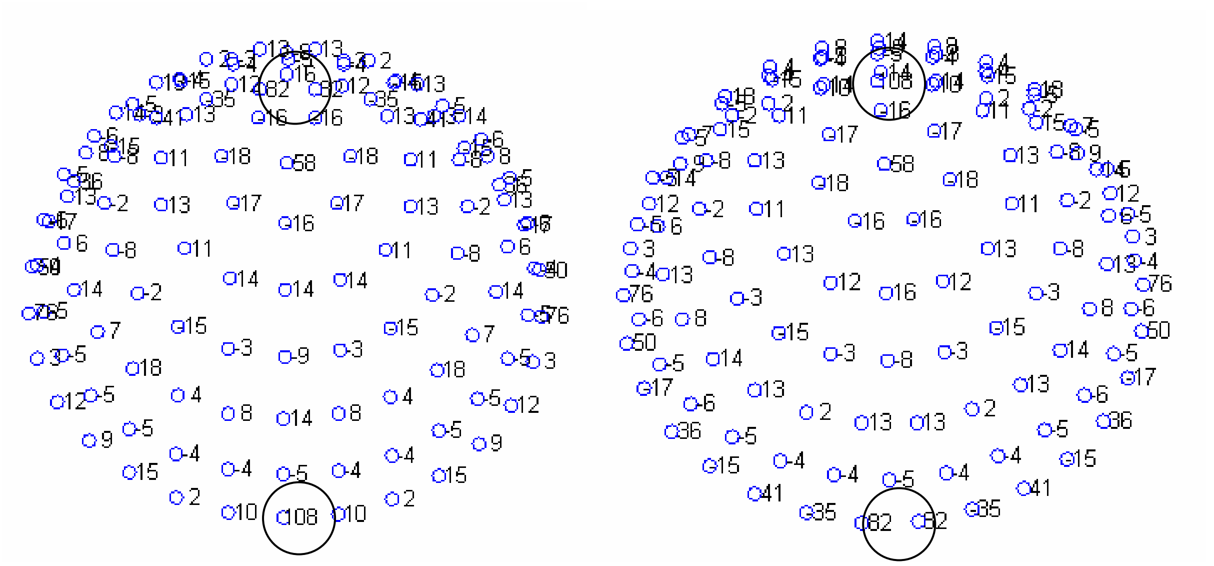

Figure 8: Tikhonov solution of the double dipole activity placed in opposite positions. The current activities are found at the expected cortex region. Both illustrations above are the same solution and the only difference is one is 180 degree reversed of each other.

4.3 2-Dipole Case: Near Dipoles

In that case, two tangential current activities are assumed to be closer. The corresponding L-curve (Figure 9) and solution (Figure 10) are given below.

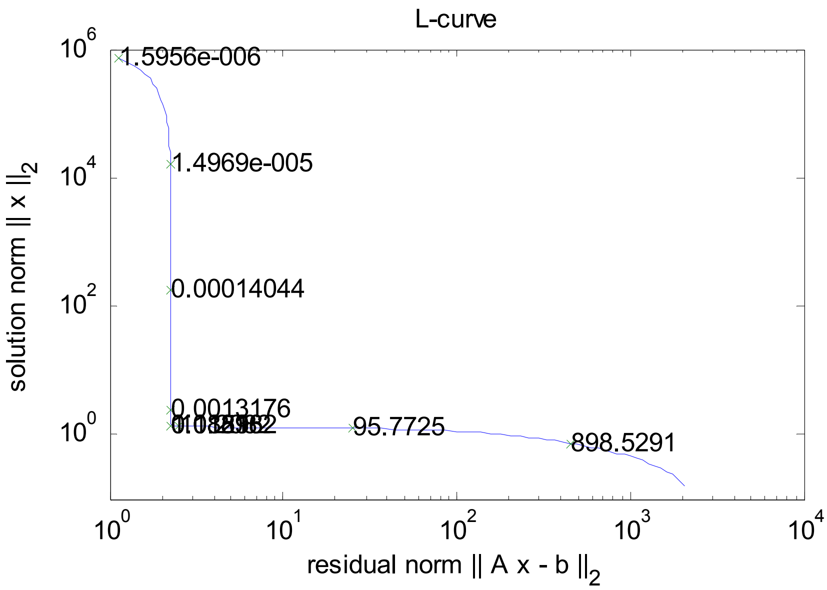

Figure 9: L-curve of the double near dipoles solution.

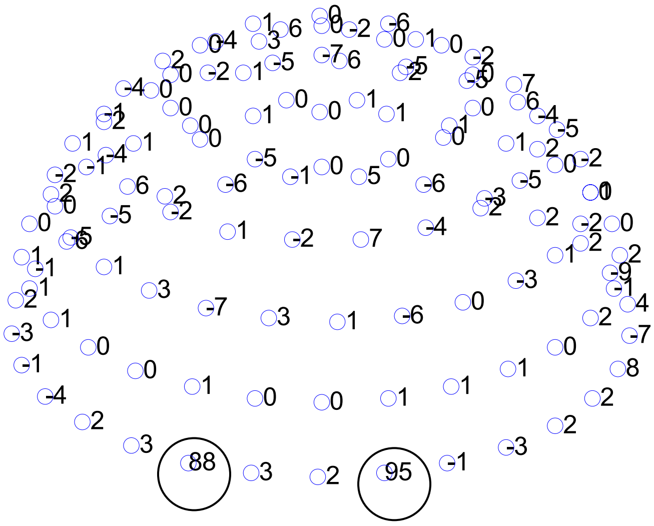

Figure 10: Tikhonov solution of the double dipole activity placed in opposite positions. The current activities are found at the expected cortex region.

5. Conclusion

Inverse problems emerge when one has indirect observations of a quantity and one seeks to make inference by computational methods of this quantity. A typical feature of inverse problems is that they are *ill-posed*: Small errors in the measured data can cause arbitrarily large errors in the estimates of the parameters of interest, or can even render the problem unsolvable. It may also occur that an inverse problem is non-unique, i.e., there are several different parameter values that could produce the same observed data. Therefore, to solve successfully inverse problems, one needs a good understanding of the questions of uniqueness and stability of the solution and methods of implementing prior information into the inverse solver algorithms.

In inverse source problems, one seeks to estimate the source of a field that is observed outside the source area. Clearly, the inverse source problems are intimately related to diffuse tomographic and wave field inverse problems as the observations are measurements of diffuse ore wave fields. Inverse source problems are encountered, e.g., in biomedical applications such as *MEG / MCG* (*Magnetoencephalography / cardiography*). Another area where these problems appear is seismology. The research in the area of inverse source problems is focused on biomedical problems, in particular, the MEG modalities.

In MEG, it is impossible to find current activities normal to the sensor orientations. Therefore only tangential dipoles are used to simulate MEG. It is tested for a single and a double current activity at the cortex and it is found that it is possible to locate these activities. It is found that it can be found a solution even if the two dipoles are so close to each other.

REFERENCES

[1] R.C. Aster, B. Borchers, and C. Thurber, *Parameter Estimation and Inverse Problems*, 2004.

[2] N.G. Gencer, S.J. Williamson, Differential characterization of neural sources with the bimodal truncated SVD pseudo-inverse for EEG and MEG measurements. IEEE Trans Biomed Eng. 45, 7 (1998) 827-838.

[3] E. Kucukaltun-Yildirim, D. Pantazis and R.M. Leahy, Signal and image processing, poster, Institute, University of Southern California, Los Angeles, CA 90089-2564.

[4] J.C. Mosher, R.M. Leahy, P.S. Lewis: EEG and MEG, Forward solutions for inverse methods, IEEE Trans on Biomed. 46, 3 (1999) 245-259.

[5] E. Somersalo, The Inverse Problem of MEG: Source localizations and the shape of a ball, SIAM News, 40, 2 (March 2007).

[6] A.N. Tikhonov and V.Y. Arsenin, *Solutions of Ill-Posed Problems*, Wiley, 1977.